Lecture 12:
Panel data II

PPHA 34600

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From last time: regression-based DD

The simplest implementation of DD is just:

$$\hat{\tau}^{DD} = (\bar{Y}(treat, post) - \bar{Y}(treat, pre)) - (\bar{Y}(untreat, post) - \bar{Y}(untreat, pre))$$

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To link these together, see:
$\bar{Y}(treat, post) = \hat{\alpha} + \hat{\tau} + \hat{\beta} + \hat{\delta}$
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Running this regression yields $\hat{\tau} = \hat{\tau}^{DD}$

Assessing the validity of the identifying assumption

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In words:

- Parallel counterfactual trends

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In different words:

- Conditional on covariates, treatment is as good as randomly assigned

In other different words:

- Treated and untreated units would be on similar trajectories if not for treatment

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In other different words:

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→ Just controlling for $Treat_i$ and $Post_t$ is usually not good enough!

→ We also need a story for why $Treat_i \times Post_t$ is quasi-random!

Fixed effects models

DD is a specific case of the fixed effects model, which lets us:

- Easily incorporate more units and time periods
- Allow for different units to be treated at different time periods
- Allow the effect of treatment to vary over time
- Enable us to more easily asssess the identifying assumptions

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- δ_t: Common time-period-specific bit
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Can we do better than leaving all of these in the error term?

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There are two main ways to do this:

1. Use dummy variables
2. "De-meaning" the data

→ These are both "fixed effects" estimators

Fixed effects estimator 1: Use dummy variables

Consider the following regression model with only individual (no time) effects:

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- Just individual-specific effects (intercepts, if you want)

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Fixed effects estimator 2: De-meaning

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We can get the same effect as adding dummies by "de-meaning" the data

- In general, define $\tilde{V}_{it} = V_{it} - \frac{1}{T}\sum_{t=1}^{T} V_{it} = V_{it} - \bar{V}_i$

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We can then express the "de-meaned" version of our model:

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$$\tilde{Y}_{it} = \tilde{X}_{it}\beta + \tau\tilde{D}_{it} + \tilde{\nu}_{it}$$

$\rightarrow$ This gives you the same result as "controlling for" α_i

We can extend these estimators to a more complex model

Consider the regression model with both individual and time effects:

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Define

$$\tilde{\tilde{V}}_{it} = V_{it} - \frac{1}{T}\sum_{t=1}^{T} V_{it} - \frac{1}{N}\sum_{i=1}^{N} V_{it} + \frac{1}{N \times T}\sum_{i=1}^{N}\sum_{t=1}^{T} V_{it}$$

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The model is now:

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Connecting FE to DD

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→ Note that we've lost α: this is now collinear with α_i

→ With just two units, α_i is just $\mathbf{1}[unit = i]$ and $\mathbf{1}[unit = j]$

→ With just two time periods, α_i is just $\mathbf{1}[time = 0]$ and $\mathbf{1}[time = 1]$

Extending DD to multiple treatment times

What happens if we have treated units who get treated at different times?

- Our new general formulation works!

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- (Or can stay 0 always for some units)
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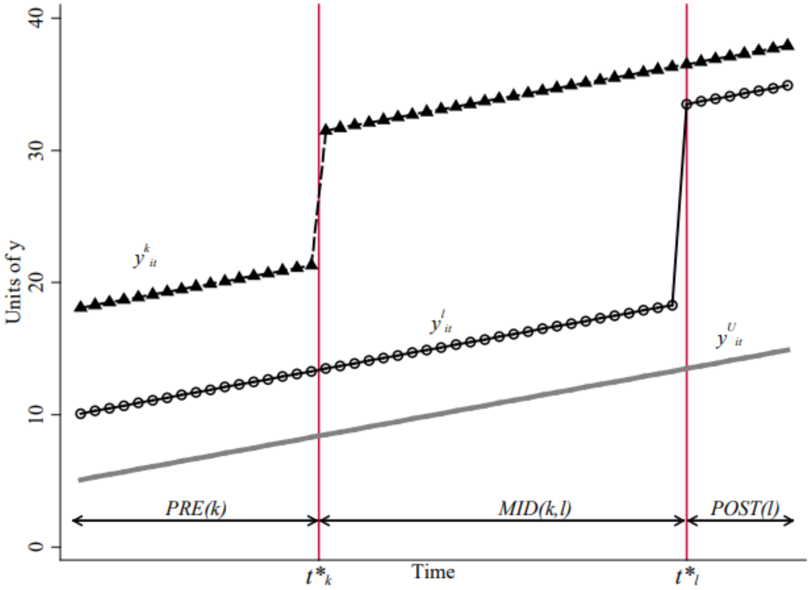
Important note: Running this specification gets you a **weighted average** of several comparisons. This may not be exactly what you want!

What does multiple-treatment-timing FE get us?



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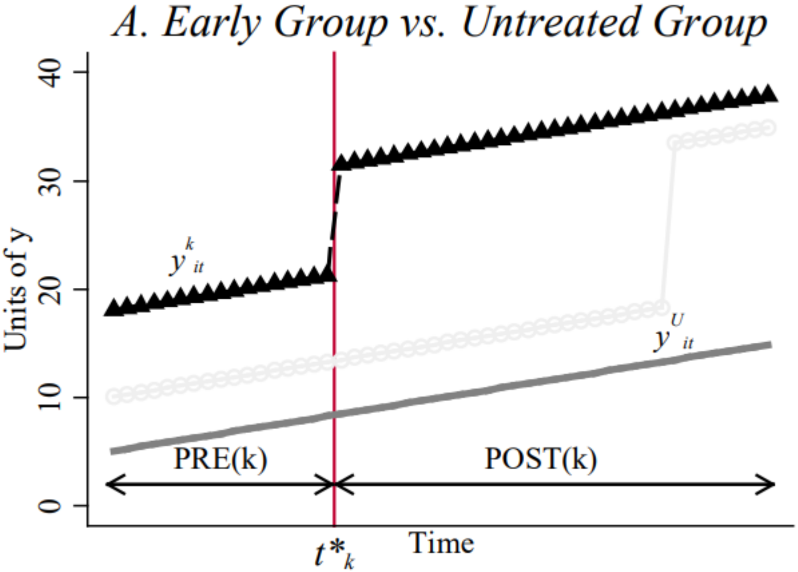

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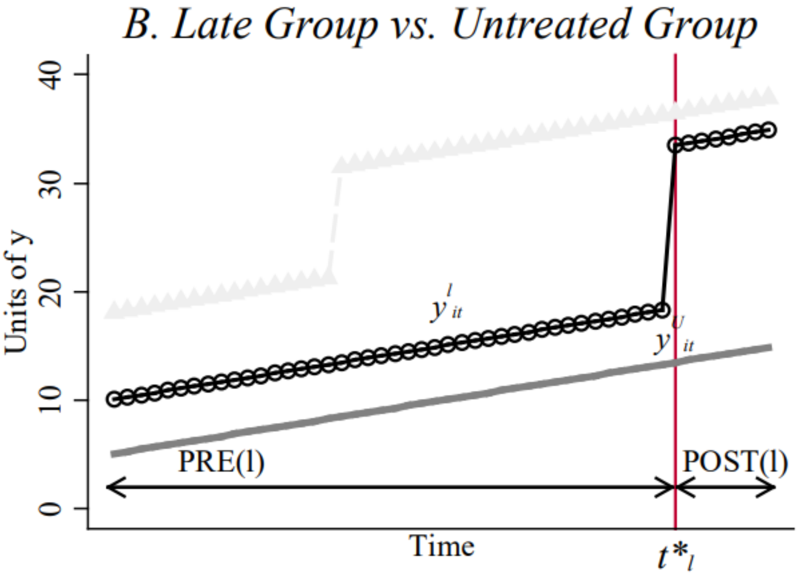

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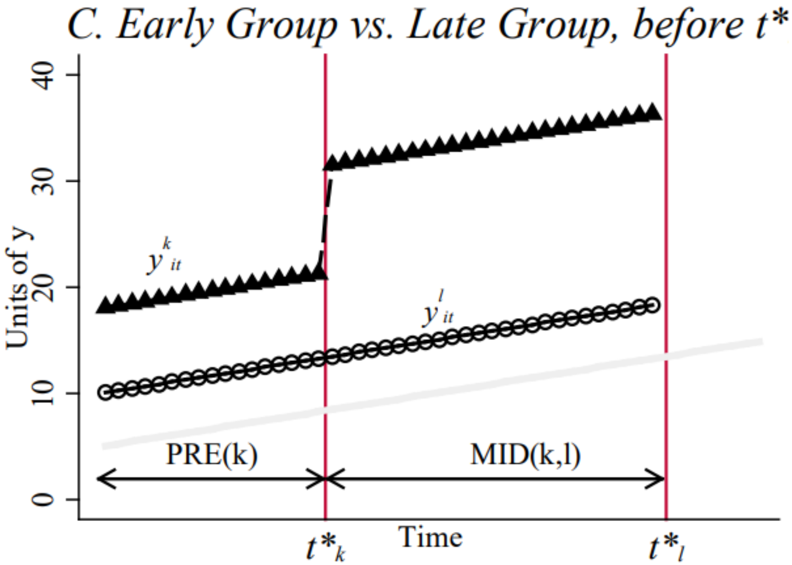

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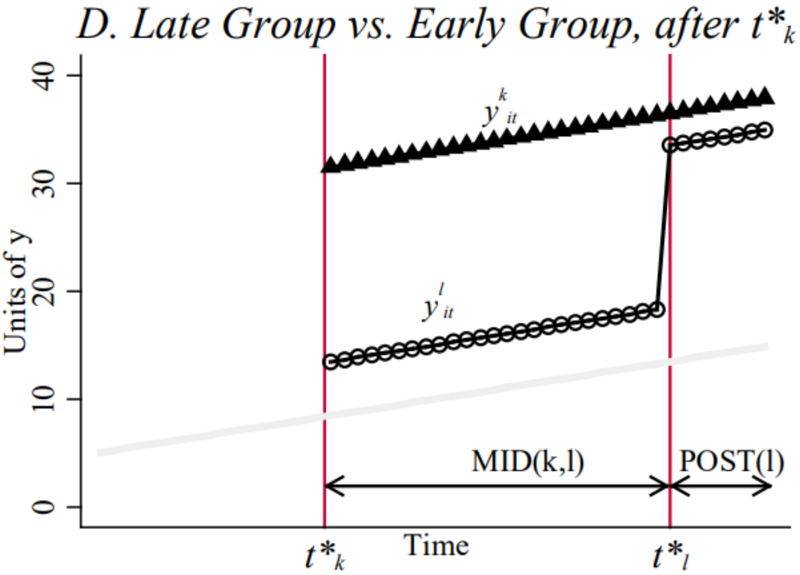

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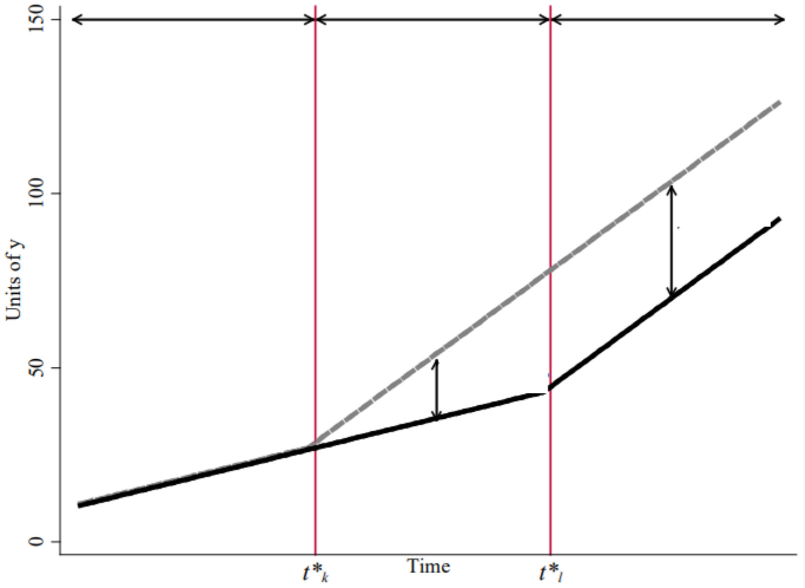

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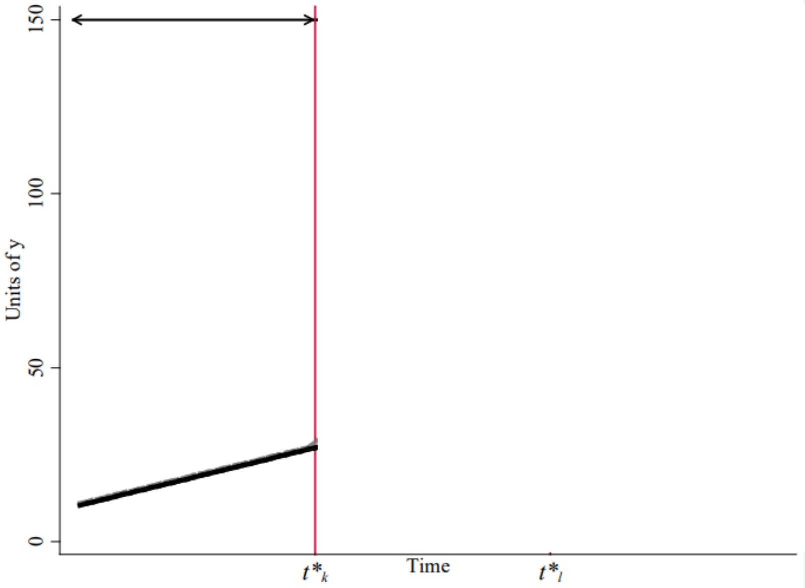

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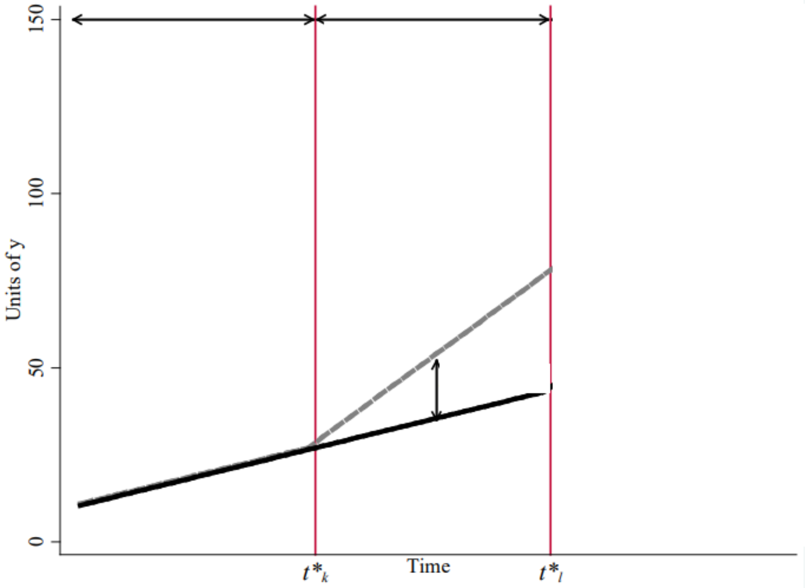

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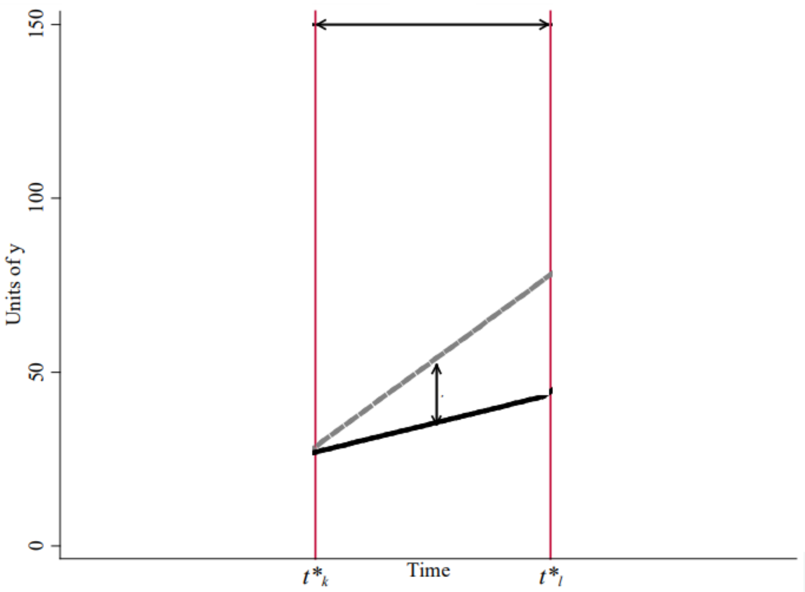

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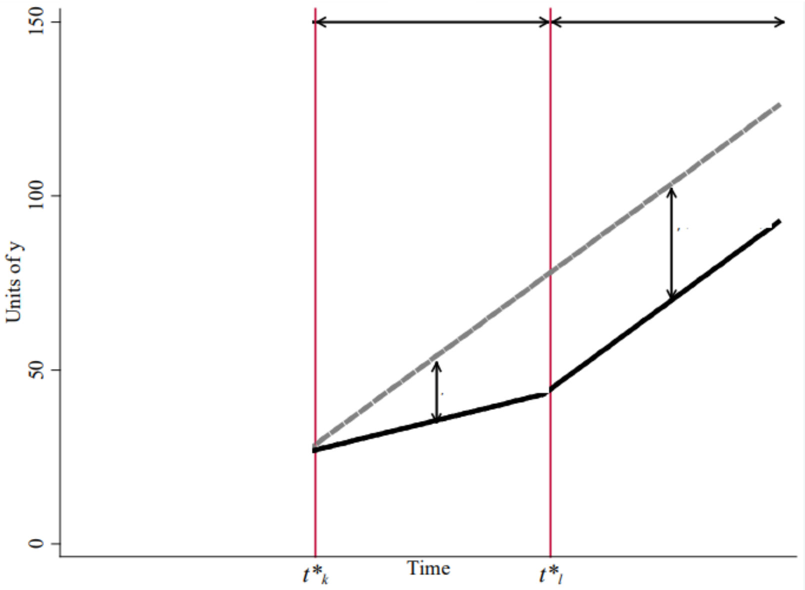

How do we do this right?

There are two main solutions to this problem:

1. Weighted balance test to make sure this isn't a problem
   - Beyond the scope of this class
2. Artificially "treat all units at the same time"

The event study design

An event study is a more general FE design:

Our standard FE regression model:

$$Y_{it} = \tau D_{it} + \alpha_i + \delta_t + \beta X_{it} + \varepsilon_{it}$$

- This imposes the constraint that $\tau_t = \tau$ for all t
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- The τ_rs pick up the average treatment effect r periods after treatment

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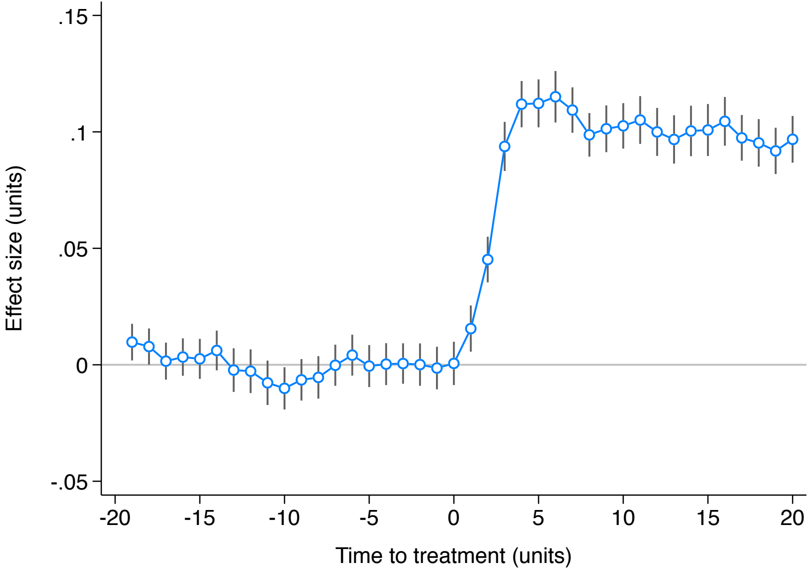
  - $\rightarrow$ We want pre-treatment τ_s's to be centered on 0 and not trending

Event studies should always be shown graphically



Cumulative effects

We're often interested in summing up effects over time:

- This is a simple extension of per-period event study effects
- And very powerful!
- We can use this to understand if treatment moves things in time
- Or happens and fades away
- Or remains important over time

Cumulative effects

We estimate cumulative effects with a distributed lag model:

$$Y_{it} = \sum_{s=0}^{S} \tau_s D_{i,t-s} + \alpha_i + \delta_t + \beta X_{it} + \varepsilon_{it}$$

where $D_{i,t-s}$ is an indicator equal to the treatment status in period $t - s$

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→ The cumulative effect q periods after treatment is:

$$T_q = \sum_{s=0}^{q} \tau_s$$

Recap

TL;DR:

1. We like the difference-in-differences approach a lot
2. We discussed estimation with fixed effects
3. And covered the event study version