Lecture 08: Instrumental variables I

PPHA 34600

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From last time: selection on observables

We tried our first non-experimental estimators:

- SOO requires **extremely strong** assumptions (☠)
- ...but when you're stuck in this world, you can use:
  1. Regression adjustment
     - Controlling for stuff
  2. Matching
     - Pairing treated and untreated on observables

→ We typically **don't** believe SOO designs

Moving on from SOO

SOO requires strong assumptions:

- "I know everything about everything"
- $(Y_i(1), Y_i(0)) \perp D_i | X_i$
- Need to isolate **all** "bad" variation in D_i

We move to selection on unobservables instead:

- "I know a little bit about a little bit"
- Typically represented as $(Y_i(1), Y_i(0)) \perp Z_i$ and $Cov(Z_i, D_i) \neq 0$
- Need to find some source of "good' variation in D_i

Where does this "good variation" come from?

We turn to natural experiments:

- Rather than needing to observe everything...
- ... we observe some (quasi) random variation in D_i
- Natural experiments are "naturally" occurring random variation
  - Can come from actual nature (e.g. variation in temperature / rainfall)
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Some new definitions:

- **Observational data:** Data generated from non-experimental settings
- **Identification strategy:** approach to using observational data to estimate causal effects
- **Identifying assumptions:** Assumptions required for the identification strategy to causally estimate impacts

Isolating good variation

Consider the following regression model:

$$Y_i = \alpha + \tau D_i + \beta X_i + \varepsilon_i$$

where:

Y_i is our outcome of interest

D_i is treatment

X_i is a set of covariates, where $Cov(X_i, \varepsilon_i) = 0$

ε_i is the error

$\rightarrow$ What formal condition do we need to recover the causal effect of D_i?

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→ What formal condition do we need to recover the causal effect of D_i?

→ We need $E[\varepsilon_i | D_i] = 0 \iff Cov(D_i, \varepsilon_i) = 0$

What happens when $E[\varepsilon_i|D_i] \neq 0$?

In this case, D_i is endogenous:

$\rightarrow$ We cannot get an unbiased estimate of τ^{ATE}

- This can result from:
  - Omitted variable bias
  - Reverse casuality (simultaneity)

Isolating exognenous variation

$$Y_i = \alpha + \tau D_i + \beta X_i + \varepsilon_i$$

Suppose we can separate D_i into two parts:

$$D_i = B_i \varepsilon_i + C_i$$

with $Cov(C_i, \varepsilon_i) = 0)$

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$\rightarrow$ ...but we can't

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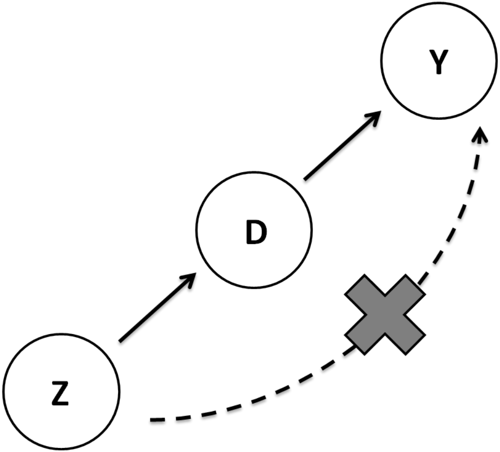
   - This is actually testable!
2. **Exclusion restriction**: $Cov(Z_i, \varepsilon_i) = 0$
   - Z_i and ε_i are **not** related
   - Z_i only affects Y_i through D_i
   - Fundamentally untestable! ☠

The exclusion restriction



Z_i affects Y_i only through D_i: Z_i cannot affect Y_i through any other channel, and therefore can be "excluded" from a regression of Y_i on D_i

Estimation of IV

Strictly speaking, the IV estimator is:

$$\hat{\tau}^{IV} = (Z'D)^{-1}(Z'Y)$$

Since we don't do matrix algebra in this class, without covariates, we can write:

$$\hat{\tau}^{IV} = Cov(Z_i, Y_i)/Cov(Z_i, D_i)$$

It's much easier to think through IV through its mechanics

Two stage least squares (2SLS)

The classic way to perform IV is via 2SLS, in several steps:

1. **First stage:**

   Regress endogenous D_i on all exogenous variables (including Z_i)

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→ Store the predicted values of D_i, $\hat{D}_i$

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  - By assumption, Z_i is unrelated to Y_i except through D_i
  - ...so a regression of D_i on Z_i only keeps the good stuff
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  - → The proper way to do this is with an F test
  - → Rule of thumb: need F statistics > 20

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  - → Rule of thumb: need F statistics > 20
- Should have an intuitive sign

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2. **Second stage:**

   Regress outcome Y_i on predicted $\hat{D}_i$ and other Xs:

   $$Y_i = \alpha + \tau \hat{D}_i + \delta X_i + \varepsilon_i$$

   → $\hat{\tau}$ in this equation is our IV estimate

   → Note: The standard errors on this will be wrong! Use a canned routine

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- **Central assumption of IV:**

→ Z_i is as good as randomly assigned!

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- You should be able to talk about the difference between this and the OLS

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The reduced form

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- This does **not** recover $\hat{\tau}^{IV}$
- Instead, this tells us how your outcome varies with the instrument
- This needs to have a causal interpretation!
  - If you don't believe that your Z_i is "as good as random" wrt Y_i, it definitely can't help you with D_i
  - In almost all cases, the RF should be independently interesting

Alternative ways to estimate τ^{IV}

We've estimated three total coefficients:

1. **First stage:** $\hat{\gamma}$ is the effect of our instrument on treatment
2. **Reduced form:** $\hat{\theta}$ is the effect of our instrument on our outcome
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→ Very Smart People realized that we can just compute:

$$\hat{\tau}^{IV} = \frac{\hat{\theta}}{\hat{\gamma}}$$

- This should be intuitive:
  - We're just *scaling* the reduced form by the first stage
  - Our IV estimate is just the effect of the instrument on our outcome, weighted by how much the instrument moves treatment
- Standard errors are still tricky (use that canned routine)

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Do we believe this? Why or why not?

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  - → IV is "throwing out" variation!
  - → You need a canned routine to get them right in any two-step approach

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  - → (The real reason why is Linear Algebra)

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- You might encounter the "forbidden regression"
  - → You need to use OLS for the first and second stage and RF
  - → No non-linear estimators!
  - → ($D_i = \alpha + \delta Z_i + \beta_1 X + \beta_2 X^2 + \varepsilon_i$ is ok)
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- **The exclusion restriction is fundamentally untestable**
  - → ☠☠☠

Recap

TL;DR:

1. Instrumental variables are very powerful
2. ...but they require extremely strong assumptions!
3. Hashtag no free lunch